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# MULTI-OBJECTIVE OPTIMIZATION OF SPACECRAFT TRAJECTORIES FOR SMALL-BODY COVERAGE MISSIONS

David Hinckley Jr.<sup>\*</sup>, Jacob Englander<sup>†</sup>, and Darren Hitt<sup>‡</sup>

Visual coverage of surface elements of a small-body object requires multiple images to be taken that meet many requirements on their viewing angles, illumination angles, times of day, and combinations thereof. Designing trajectories capable of maximizing total possible coverage may not be useful since the image target sequence and the feasibility of said sequence given the rotation-rate limitations of the spacecraft are not taken into account. This work presents a means of optimizing, in a multi-objective manner, surface target sequences that account for such limitations.

## INTRODUCTION

When conducting a small-body mission there is often the need to develop a detailed map of the surface. Topography-based position tracking and landing operations have direct need of detailed information regarding the body's surface. To acquire such information survey trajectories must be flown in which images of the surface are taken which must adhere to a standard of coverage in order to learn all of the necessary facets of the surface. During survey trajectories a camera on-board the spacecraft must take the proper sequence of images, focusing on the surface at specific locations at specific times. The determination of this image sequence for a given trajectory is not a trivial matter as both coverage achieved and rotational effort expended are potentially opposed. This work optimizes the target viewing sequence for a sample trajectory considering both objectives using multi-objective evolutionary algorithm approach.

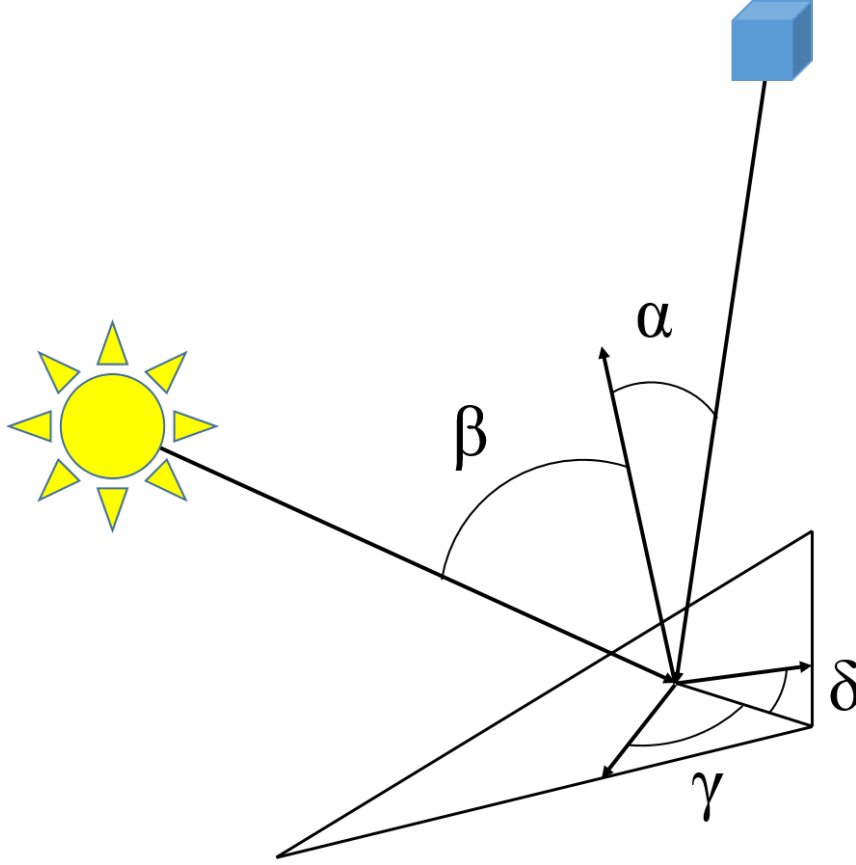
Coverage of a surface site is defined by a combination of images taken meeting different requirements placed upon relative angles of the sun and camera. For any site, the emission angle is defined as the angle between the surface normal and the sun's line-of-sight (LoS) to the site. Coverage requires images with at least three different emission angles within the bounds of  $[20^\circ, 60^\circ]$  mutually separated by  $8^\circ$ . Incidence angle is defined as the angle between the surface normal and the spacecraft's LoS. Coverage requires images with at least three different incidence angles within the bounds of  $[20^\circ, 70^\circ]$  mutually separated by  $10^\circ$ . Spacecraft azimuth angle is the angle between a surface projection of the spacecraft's LoS and a surface reference. Coverage requires images with at least three different spacecraft azimuth angles mutually separated by  $90^\circ$ . Solar azimuth angle is defined as the angle between the surface projection of the Sun's LoS and a surface reference. Coverage requires images with at least two different solar azimuth angles separated by  $150^\circ$ . A visual depiction of these angles is given by Figure 1. Coverage also requires ten unique observations

<sup>\*</sup>Mechanical Engineering Graduate Student, University of Vermont

<sup>†</sup>Aerospace Engineer, Navigation and Mission Design Branch, NASA Goddard Space Flight Center

<sup>‡</sup>Professor of Mechanical Engineering, University of Vermont

which are defined as unique combinations of emission, incidence, and spacecraft azimuth angles. These requirements are currently used by NASA for small-body missions.

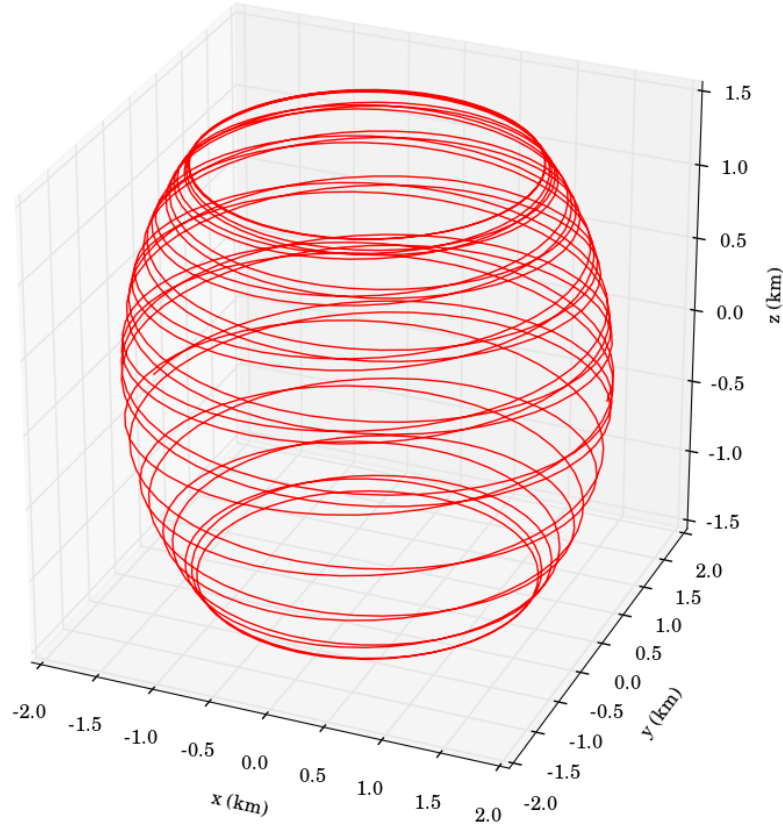


**Figure 1. Definition of angles: Emission angle  $\alpha$ , Incidence angle  $\beta$ , Spacecraft Azimuth angle  $\gamma$ , Solar Azimuth angle  $\delta$**

## PROBLEM DESCRIPTION

The test problem for this work uses the asteroid Bennu as the body of interest with a  $45^\circ$  inclined “circular” (the velocity was established using two-body relations) surveying trajectory set at a distance of 2 km from Bennu’s center of mass. The gravity calculation was performed using a mason method<sup>1</sup> and the surface model of Bennu required for coverage computation was found online.<sup>2</sup> Figure 2 shows the trajectory created by the non-spherical mass-distribution. It is this non-Keplerian orbit that allows for many of the coverage requirements to be met; a simple elliptical closed orbit would not provide the sufficient viewing angles within the same time frame.

The camera half-angle used is  $2^\circ$ . The length of trajectory considered is chosen as 5 days, as it is neither prohibitively long nor too short, with image opportunities every five minutes so as to allow ample time to achieve any desired orientation. The total coverage possible for this trajectory, found by using every target at every location, is 25.2%. Figure 3 shows the faces that could possibly be covered for this test trajectory. As seen, only near equatorial faces can be covered due to the relative orientation of the sun; many faces never achieve the solar-azimuth requirement which is independent of trajectory but still dependent on the length of time considered. This test problem



**Figure 2. Survey trajectory representation**

is simplified such that the Sun's line of action is treated as constant due to the short length of time considered; the exact direction within the equatorial plane is also not considered significant due to the difference in scale of the rotation rate of the asteroid relative to its orbit.

For this problem solution takes the form of a list of integers of length equal to the number of viewing opportunities with each entry corresponding to a face in the asteroid surface model. With opportunities every five minutes, this means that each solution has 1440 targets in sequential list form. For this type of problem, two objectives are of particular importance thus the problem must be addressed in a multi-objective fashion. The two objectives considered are the maximization of achieved coverage and the minimization of the required change in rotation rate, slew rate, required to accomplish the viewing schedule.

## METHODS

The viewing target sequences are optimized using the Multi-Objective Evolutionary Algorithm (MOEA) Non-dominated Sorting Genetic Algorithm 2 (NSGA-2).<sup>3</sup> NSGA-2 works by lexicographically rank-sorting a population of solutions first by non-domination rank and then by crowding distance. Non-domination rank is the number of solutions that dominate (i.e. that are more fit in every objective) the given solution. Crowding distance is a numerical representation of the proximity of the given solution to its neighbors in the objective space. This sorting occurs each generation and trends the population toward improvement by its preferential inclusion of the best of a generation into the next generation. The genetic operators used are uniform cross-over and element-wise uniform random mutation. A 1-point and 2-point cross-over were tested but showed slower progression of the non-dominated front through the objective space. This is thought so since fitness does not have a strong coupling with target sequence because the time scale of image opportunities is such that switching targets locally in time does not appreciably affect fitness.

Using these operators, the NSGA-2 implementation evolves target sequences under the consideration of the base objectives of maximizing achieved coverage (framed as minimizing the additive inverse of achieved coverage) and minimizing the normalized (unity moment of inertia for each axis) change in rotation rate (summed across three orthonormal axes) required by the schedule. It is in these operators in which stochasticity is leveraged. The mutation operator operates probabilistically on each genome location with a probability of  $1/L$  where  $L$  is the length of the genome; thus for all problem lengths the probability of no mutation is 37%. The crossover operator, in this uniform implementation, probabilistically chooses between two parents for each genome location, each location chosen independently.

The emission angle, incidence angle, and spacecraft azimuth angle requirements are handled using a multi-layered “binning” of observations for each face. By “binning” it is meant that the angular domain is discretized into sections with widths equal to the desired separation for each requirement, with a binary value corresponding to each section representing whether an observation has been made within that sub-domain. If only a single bin-layer were used, two observations which differed by a fraction of a degree could straddle the boundary of a sub-section and cause that which is essentially one observation to count as two toward the requirements. Additional layers, each rotated relative to one another prevent this. The introduction of a second layer of the same discretization width yet rotated half the separation angle prevents the aforementioned case but the failure case extends in the new implementation to two pairs of observations, with each pair basically indistinguishable, being counted as four observations still skirting the requirement. Thus for  $n$  independent observations separated a fixed angle,  $n$  layers each rotated  $\frac{1}{n}^{th}$  the separation angle are required since the failure case morphs to  $n$  unique observations registering as  $2n$  yet since only  $n$  are required the condition is still met. For each face, once each layer has the required number of activated bins the requirement is met. The uniqueness requirement, which pertains to combinations of the above requirements, is handled using a super-increasing list implementation which uniquely maps the combinations of the first layer of activated bins to integers; when a collection of ten different integers is established for a face, then the requirement is met for that face. The solar-azimuth constraint is handled by saving new observations and comparing combinations; this process is stopped for a face once the condition is met.

## RESULTS

Figure 4 shows a non-dominated front that is the result of 1750 generations of evolution with a population size of 30 candidate solutions. Since the asteroid's surface is discretized, the percent area covered also assumes discrete values. Although the front shows not much breadth in terms of coverage fraction, 96.22% of the maximum possible coverage was achieved. The lack of varying coverage fractions indicates the problem is such that there exist many sequences that achieve the same, relatively high, coverage. If one were to consider a candidate solution in which the spacecraft did not rotate, such solution would have 0 deg/s rotation rate change and thus be non-dominated and appear on the final front. Yet such a solution is not possible with the current problem transcription since orientation in space is not specified at each opportunity. Instead faces to target are specified and since the model dictates their dispersion in space, there is not necessarily, as is true here, the option to continue with the rotation rate of the prior transition. This means that the choice of problem transcription, as is always inherently true, limits the attainable solution set and precludes some solutions to the intended problem that may be found via inspection. If more generations were pursued, or a population size greater than 30 were used, then a more resolved front could be achieved if such were possible for this problem.

Beyond a pragmatic limit, continuing the evolution has diminishing returns. To illustrate this, Figure 5 shows the same solution set after only 1000 generations of evolution and Figure 6 shows the same for 500 generations. Aside from the narrow spread of solutions over local quantized coverages, it is evident that the front is quite linear in form. It is also evident that progress of later generations reduces to fractions of that achieved in previous generations. This may be used to specify an early termination condition based upon insufficient progress in any single objective or upon the lack of motion of the closest objective point that dominates the entire set.

One important aspect of coverage that is not addressed in this optimization is the desire to maximize the coverage return per unit time, that is to observe the potential trades when time-to-coverage is also minimized. This means that one might find sequences such that the entire trajectory may be shortened by days by sacrificing perhaps a few percent of the coverage achieved. Figure 7 shows a time-to-coverage heat map for the solution of the front with the lowest required change in rotation rate. From this one sees that the time in which a face is covered is dependent on both its orientation (the observed local fluctuations) and also on its position on the body, as seen in the primarily equatorial region of coverage.

During testing it was discovered that the solar-azimuth angle requirement is often a limiting factor in the coverage achieved. If this requirement were to be ignored, the maximum possible coverage increases from 25.2% to 78.5%. Figure 8 shows the non-dominated front of a different evolution test in which the solar-azimuth requirement was ignored; simply reviewing the prior results with the requirement changed would not be the same since the evolution may have progressed differently with the requirement not being considered. Both the coverage and required change in rotation rate outclass the previous result shown for the same number of generations of evolution. The lack of variety in solar-azimuths observed relates to sole spin-axis of the asteroid being orthogonal to the Sun-Bennu line (as represented in this test problem). Due to the permanent orthogonality in this test problem, the solar-azimuths observed for locations near the poles do not span the necessary  $150^\circ$  thus the requirement of having two images taken with a greater separation cannot be met.

## CONCLUSION

In this work, we used a multi-objective Evolutionary Algorithm to optimize the surface target viewing sequence for an example mission survey trajectory about the asteroid Bennu. NSGA-2 was used to optimize the sequences since its operators are well suited to finite-alphabet problems such as this one. The results were shown to recover 96.22% of the total possible coverage while minimizing the associated required change in rotation rate. While the non-dominated fronts had not much extent in terms of coverage, with all of the population members exhibiting the same high coverage values, this is more the consequence of the problem description and transcription than the optimization. It was shown that a severely limiting factor in terms of coverage was the solar-azimuth constraint which precluded the possibility of coverage for over 50% of the asteroid's total surface. If the spin axis of the body of interest were not orthogonal to the Sun-body line, then the requirement would be more readily met.

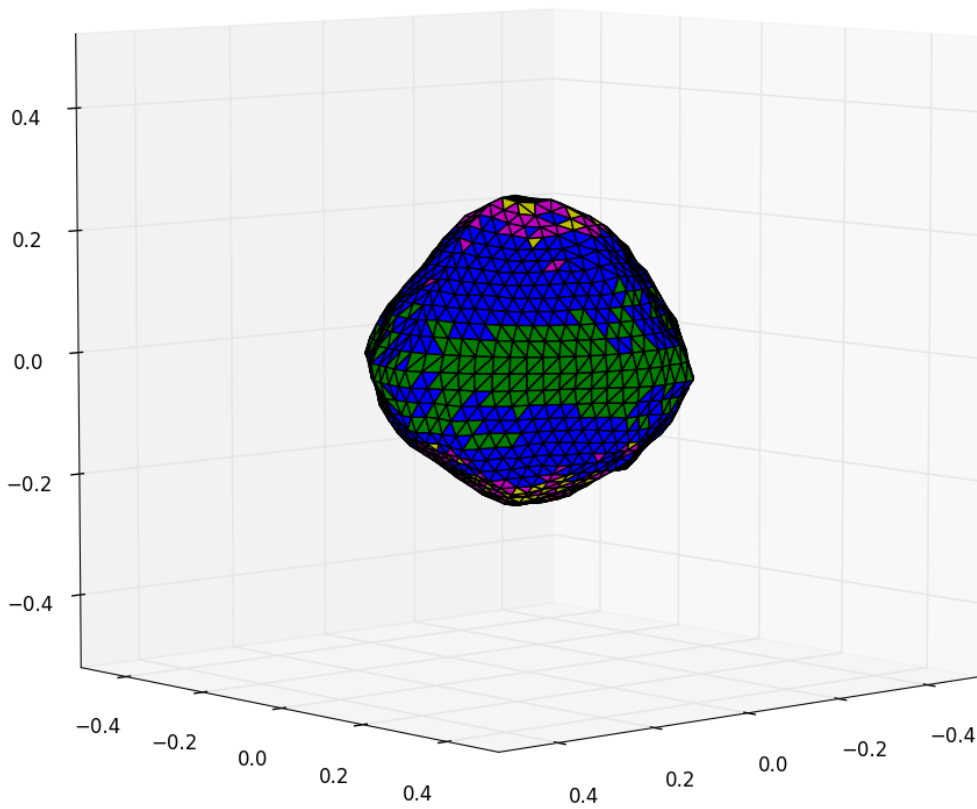
## Future Work

Currently the coverage objective value is the coverage itself but this means that small advantageous genetic changes that condition better coverage later in evolution are not numerically appreciated until that coverage is realized. It may then hold merit to optimize a function analogous to coverage which rewards achieving any of the sub-requirements to coverage and not just coverage itself. The danger in this is the potential to favor the accrual of points for sub-requirement satisfaction of many surface elements instead of achieving coverage of fewer faces, the latter being preferable. The balancing measures taken against such local optima, taken under the hope that this new fitness definition may converge to the non-dominated front faster, may prove to hinder or ill-condition progress such that the old fitness definition proves more effective; but such an investigation is worthwhile.

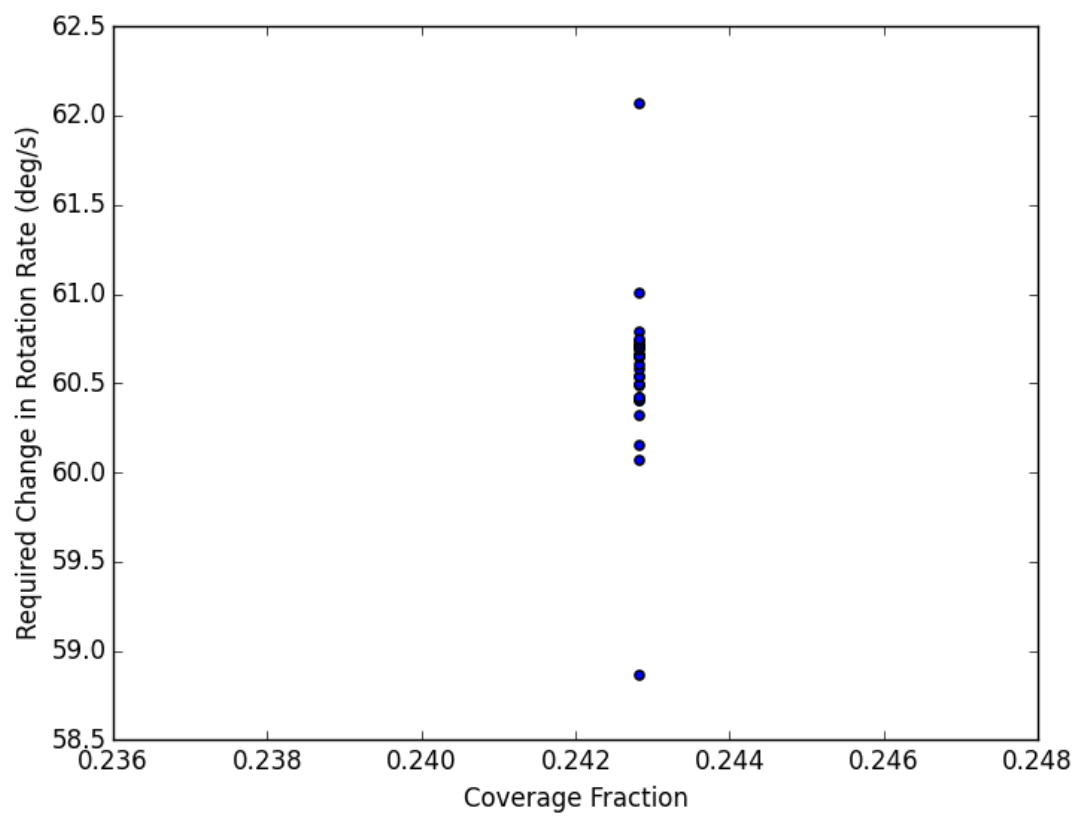
The optimization of viewing sequences is a sub-task of the full incarnation of this approach which would also optimize the survey trajectories themselves based on the results of the viewing sequence optimization with additional considerations made for fuel expenditure and bounded orbital parameters. This would be hybrid optimal control framework in which both the inner and outer loops are multi-objective;<sup>4</sup> this could be handled by a single multi-objective optimizer in which a candidate solution is both the trajectory and viewing sequence but this would lead to duplicate evaluation of trajectories which will be computationally the most expensive part of the process. In that framework a body in interest could be selected and an optimal viewing sequence and associated trajectory would be found. Such would require the use of a high-fidelity gravity model due to the close proximity of the spacecraft to potentially irregularly shaped bodies which presents the challenge of minimizing the necessary function evaluations to counteract the expensive integration that accompanies fitness evaluation.

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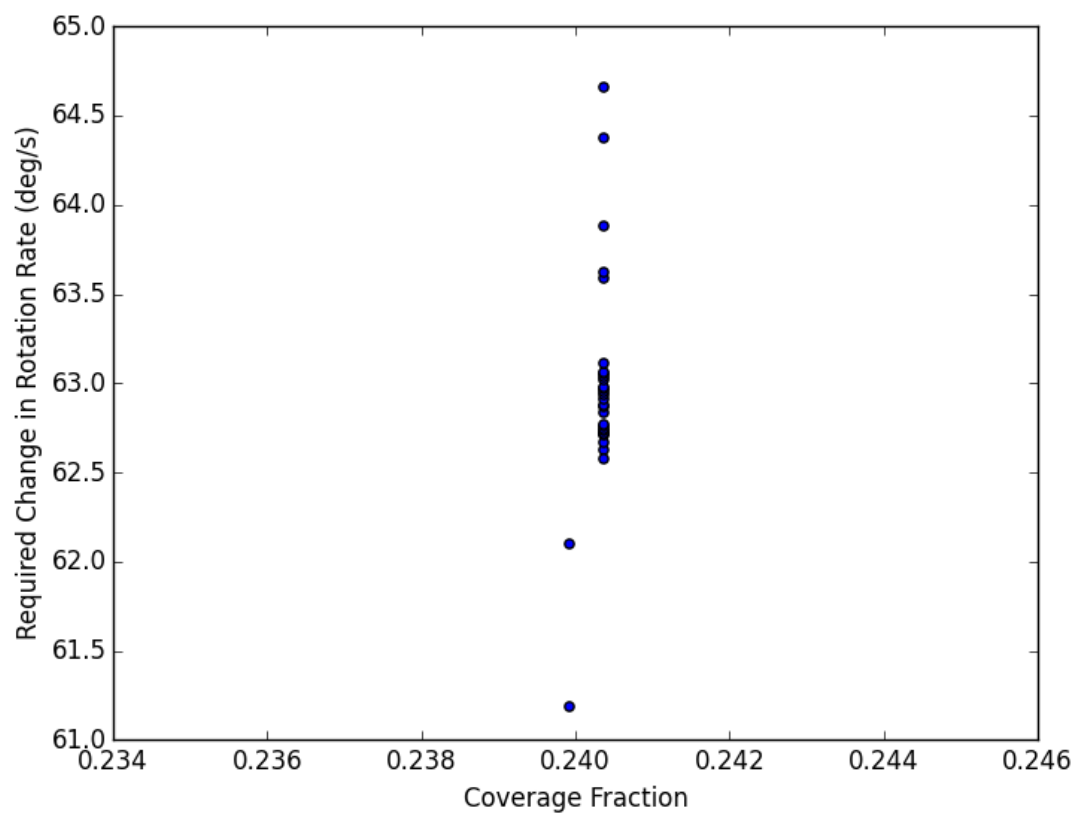


**Figure 3. Possible achievable coverage map. Color coding: green-all requirements met, blue - 1 requirement not met, purple - 2 requirements not met, yellow - 3 requirements not met, red - 4 requirements not met, black - no requirements met**

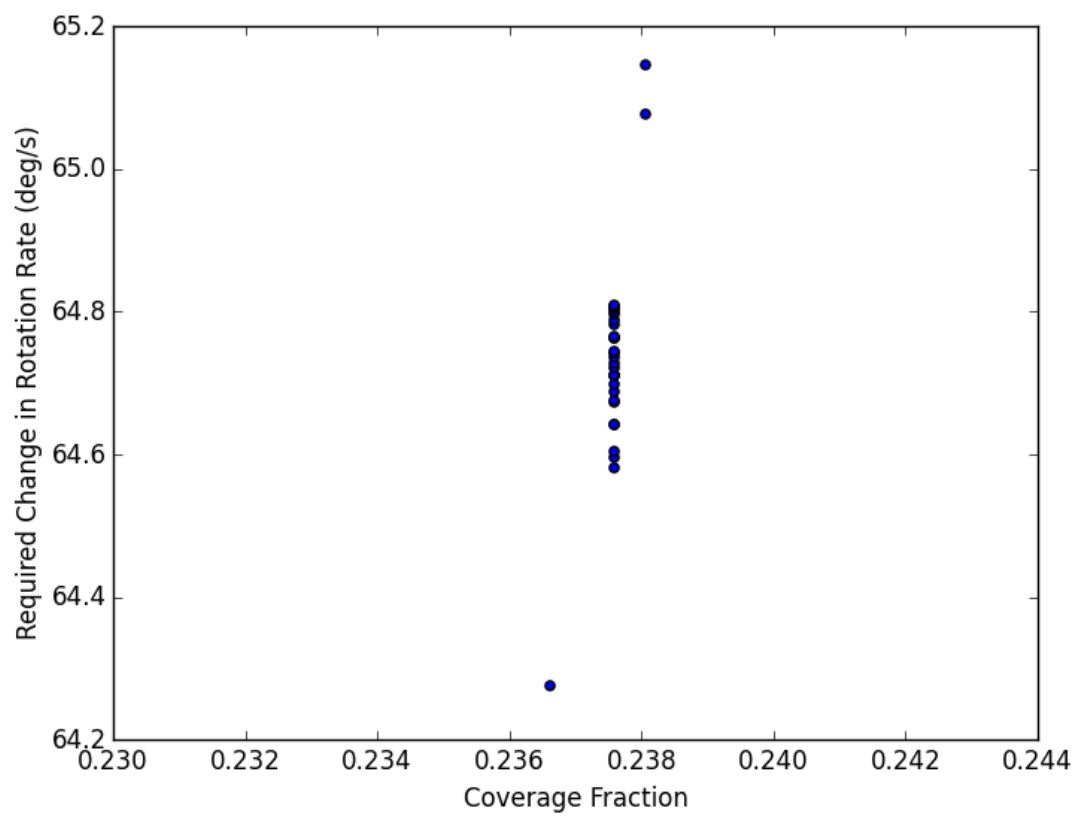


**Figure 4. Non-dominated front after 1750 generations**

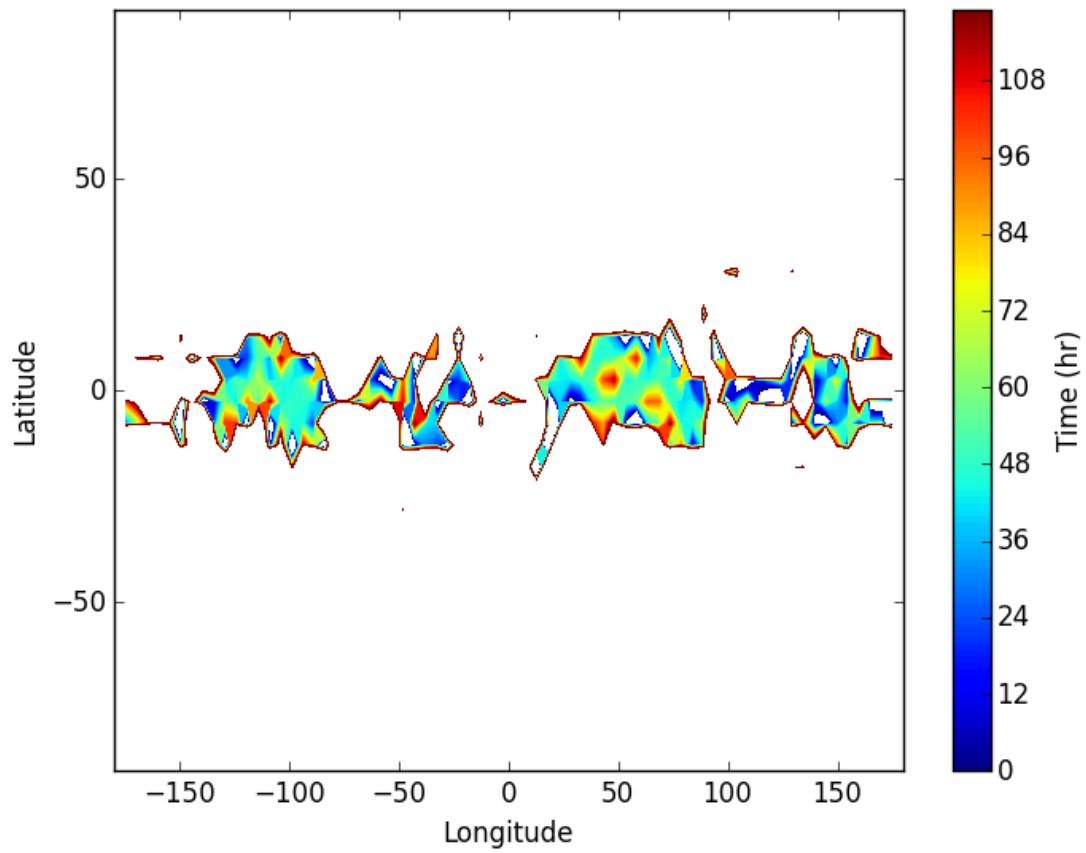




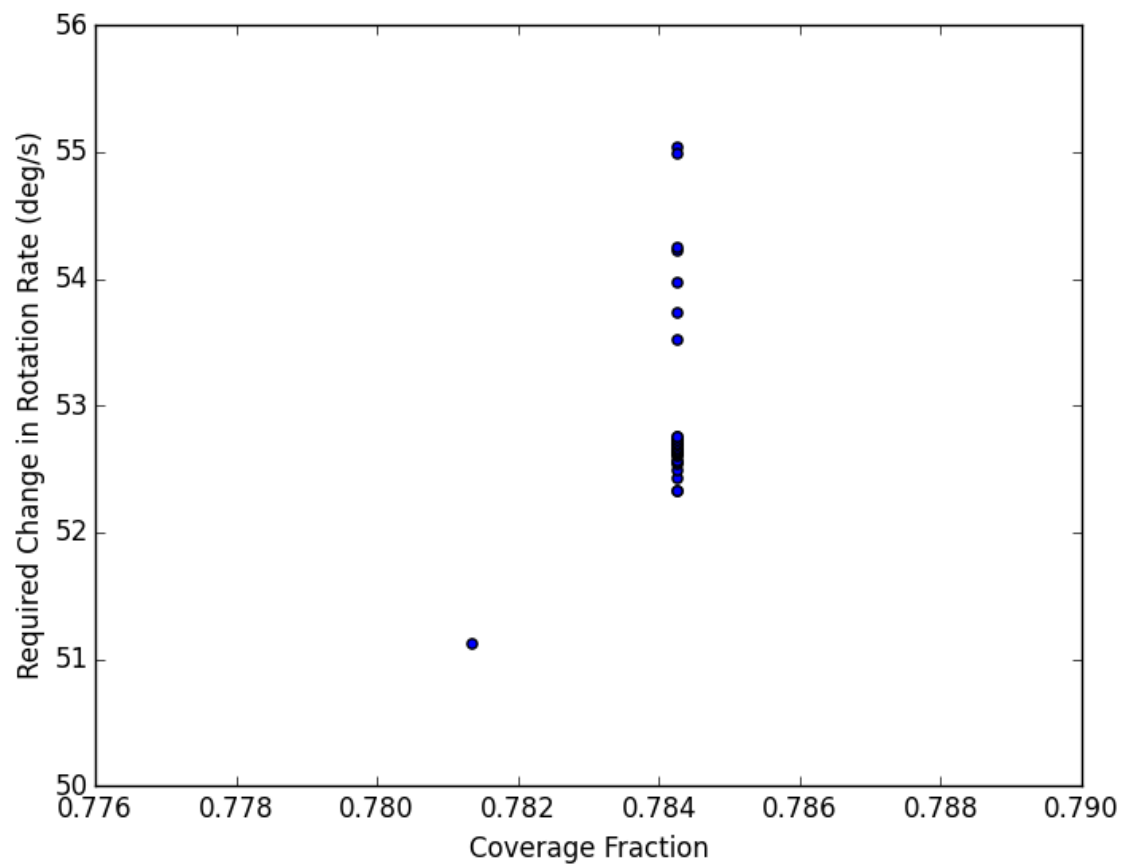
**Figure 5. Non-dominated front after 1000 generations**



**Figure 6. Non-dominated front after 500 generations**



**Figure 7. Time to Coverage Heat Map**



**Figure 8. Non-dominated front after 1750 generations**